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Intuitionistic fuzzy g - closed sets

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Abstract:- In this paper, we introduce and study the notions of intuitionistic fuzzy g -closed sets and intuitionistic fuzzy g -open sets and study some of its properties in Intuitionistic fuzzy topological spaces.

[0]**Keywords and Phrases:-** Intuitionistic fuzzy topology, Intuitionistic fuzzy g - closed sets and Intuitionistic fuzzy g-open sets.[0]2010 Mathematics Subject Classi cation: 54A40, 03F55.

I. INTRODUCTION

In 1965, Zadeh [12] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notions. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of intuitionistic fuzzy g -closed sets and intuitionistic fuzzy g -open sets and study some of its properties in intuitionistic fuzzy topological spaces.

II. PRELIMINARIES

Throughout this paper, (X;) or X denotes the intuitionistic fuzzy topological spaces (brie y IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively.

We recall some basic de nitions that are used in the sequel.

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De nition 2.1. [1] Let X be a non-empty xed set. An intuitionistic fuzzy set A in X is an object having the form

A = fhx; $_A$(x); $_A$(x)i : x 2 Xg ;

where the functions $_A$: X ! [0; 1] and $_A$: X ! [0; 1] denote the degree of membership $_A$(x) and the degree of non-membership $_A$(x) of each element x 2 X

to the set A respectively and	0$_A$(x) + $_A$(x) 1	for each	x 2 X:
De nition 2.2. [1] Let	A and B be	IFS's	of the form
A = fhx; $_A$(x); $_A$(x)i : x 2	Xg and B = fhx; $_B$(x); $_B$(x)i : x 2 Xg : Then		

1. A B if and only if $_A$(x) $_B$(x) and $_A$(x) $_B$(x) for all x 2 X:
2. A = B if and only if A B and B A:
3. A^c = fhx; $_A$(x); $_A$(x)i : x 2 Xg :
4. A \ B = fhx; $_A$(x) ^ $_B$(x); $_A$(x) _ $_B$(x)i : x 2 Xg :
5. A [B = fhx; $_A$(x) _ $_B$(x); $_A$(x) ^ $_B$(x)i : x 2 Xg :
6. 0~ = fhx; 0; 1i : x 2 Xg and 1~ = fhx; 1; 0i : x 2 Xg :
7. $1_\sim^c$ = 0~ and $0_\sim^c$ = 1~:

Denition 2.3. [3] An intuitionistic fuzzy topology (IFT) on X is a family of IFS's in X satisfying the following axioms.

1. 0~; 1~ 2 ;
2. G_1 \ G_2 2 for any G_1; G_2 2 ;

	^{S} G_i 2 for any family	fG_i : i 2 Jg :	
In this case the pair	(X;)	is called an intuitionistic fuzzy topological	space

(IFTS) and any IFS in		is known as an intuitionistic fuzzy open set (IFOS) in X.			
The complement A^c of an IFOS A in IFTS (X;) is called an intuitionistic					fuzzy
closed	set (IFCS) in X.				

We simply write A = hx; $_A$; $_A$i instead of A = fhx; $_A$(x); $_A$(x)i : x 2 Xg in case there is no chance for confusion.

De nition 2.4. [3] Let (X;) be an IFTS and A = hx; $_A$; $_A$i be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are de ned by

S int(A) = fG : G is an IFOS in X and G Ag ; cl(A) = ^{T} fK : K is an IFCS in X and A Kg :

Denition 2.5. [5] An IFS A = hx; $_A$(x); $_A$(x)i in an IFTS (X;) is said to an

1. intuitionistic fuzzy semi-open set (IFSOS) if A cl(int(A));
2. intuitionistic fuzzy pre open set (IFPOS) if A int(cl(A));
3. intuitionistic fuzzy -open set (IF OS) if A int(cl(int(A)));
4. intuitionistic fuzzy regular open set (IFROS) if A = int(cl(A));
5. intuitionistic fuzzy -open set (IF OS) if A cl(int(cl(A))):

An IFS A is said to be an intuitionistic fuzzy semi-closed set (IFSCS), intuitionistic fuzzy pre closed set (IFPCS), intuitionistic fuzzy -closed set (IF CS), intuitionistic fuzzy regular closed set (IFRCS) and intuitionistic fuzzy -closed set (IF CS) if the complement of A is an IFSOS, IFPOS, IF OS, IFROS and IF OS respectively.

Denition 2.6. An IFS A = hx; $_A$(x); $_A$(x)i in an IFTS (X;) is said to an

1. intuitionistic fuzzy generalized closed set (IFGCS) [10] if cl(A) U whenever A U and U is an IFOS in X,
2. intuitionistic fuzzy regular generalized closed set (IFRGCS) [9] if cl(A) U whenever A U and U is an IFROS in X,
3. intuitionistic fuzzy generalized semi-closed set (IFGSCS) [7] if scl(A) U whenever A U and U is an IFOS in X,
4. intuitionistic fuzzy -generalized closed set (IF GCS) [6] if cl(A) U whenever A U and U is an IFOS in X,
5. intuitionistic fuzzy generalized -closed set (IFG CS) [8] if cl(A) U whenever A U and U is an IF OS in X,
6. intuitionistic fuzzy generalized semipre closed set (IFGSPCS) [4] if spcl(A) U whenever A U and U is an IFOS in X.

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An IFS A is said to be an intuitionistic fuzzy generalized open set (IFGOS), intuitionistic fuzzy regular generalized open set (IFRGOS), intuitionistic fuzzy generalized semi-open set (IFGSOS), intuitionistic fuzzy -generalized open set (IF GOS), intuitionistic fuzzy generalized -open set (IFG OS) and intuitionistic fuzzy generalized semipre open set (IFGSPOS) if the complement of A is an IFGCS, IFRGCS, IFGSCS, IF GCS, IFG CS and IFGSPCS respectively.

De nition 2.7. [11] Let ; 2 [0; 1] with + 1: An intuitionistic fuzzy point (brie y IFP), written as $p_{(;)}$; is de ned to be an IFS of X given by

$p_{(;)}$(x) = (;) if x = p, = (0; 1) otherwise.

We observe that an IFP $p_{(;)}$ is said to belong to an IFS A = hx; $_A$(x); $_A$(x)i ; denoted by $p_{(;)}$ 2 A if $_A$(x) and $_A$(x):

De nition 2.8. [5] Two IFS's A and B are said to be q-coincident (brie y A q B) if and onlf if there exists an element x 2 X such that $_A$(x) > $_B$(x) or
$_A$(x) < $_B$(x)

De nition 2.9. [5] Two IFS's A and B are said to be not q-coincident (brie y e (A q B)) if and onlf if A B^c:

De nition 3.1. An IFS A of an IFTS	(X;)	is said to be an intuitionistic fuzzy			
g -closed set (brie y IFG CS) if	cl(A)		int(cl(U)) whenever A		U and

U is an IF OS in (X;):
Example 3.2. Let X = fa; bg and = f0$_\sim$; G; 1$_\sim$g be an IFTS on X, where

G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS A = hx; (0:4; 0:5); (0:6; 0:5)i is an IFG CS in (X;): Theorem 3.3. Every IFCS is an IFG CS but not conversely.
Proof. Let A U; where U is an IFROS. Then cl(A) cl(A) = A U: Hence A is an IFG CS. Example 3.4. Let X = fa; bg and = f0$_\sim$; G; 1$_\sim$g be an IFTS on X, where
G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS A = hx; (0:4; 0:5); (0:6; 0:5)i is an IFG CS but not an IFCS in (X;):

Theorem 3.5. Every IFRCS is an IFG CS but not conversely.
Proof. Since every IFRCS is an IFCS, the proof follows from Theorem 3.4.
Example 3.6. Let X = fa; bg and = f0$_\sim$; G; 1$_\sim$g be an IFTS on X, where
G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS A = hx; (0:4; 0:5); (0:6; 0:5)i is an IFG CS but not an IFRCS in (X;):

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Theorem 3.7. Every IF CS is an IFG CS but not conversely.		
Proof .		Let A be an IF CS and U be an IFROS such that AU: Then
cl(A)		U: Since cl(A) = A and hence A is an IFG CS.

Example 3.8. Let X = fa; bg and = f0$_\sim$; G; 1$_\sim$g be an IFTS on X, where
G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS A = hx; (0:4; 0:5); (0:6; 0:5)i is an IFG CS but not an IF CS in (X;):

Theorem 3.9. Every IFGCS is an IFG CS but not conversely.
Proof. Let A be an IFGCS and U be an IFROS such that A U: Since every IFROS is an IFOS and cl(A) cl(A); we have by hypothesis, cl(A) cl(A) U and hence A is an IFG CS.
Example 3.10. Let X = fa; bg and = f0$_\sim$; G; 1$_\sim$g be an IFTS on X, where
G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS A = hx; (0:4; 0:5); (0:6; 0:5)i is an IFG CS but not an IFGCS in (X;):
Theorem 3.11. Every IFRGCS is an IFG CS but not conversely.
Proof. Let A be an IFRGCS and U be an IFROS such that A U: Sincecl (A) cl(A) and cl(A) U; by hypothesis, A is an IFG CS.
Example 3.12. Let X = fa; b; cg and = f0$_\sim$; G_1; G_2; 1$_\sim$g be an IFTS on X, where

G_1 = hx; (0:4; 0:4; 0:5); (0:4; 0:4; 0:4)i and G_2 =				hx; (0:2; 0:3; 0:5); (0:5; 0:5; 0:5)i :		
Then the IFS A =	h	x; (0:4; 0:3; 0:2); (0:5; 0:4; 0:5)	i	is an IFG	CS but not an	

IFRGCS in (X;):

Theorem 3.13. Every IF GCS is an IFG CS but not conversely.

Proof. Let A be an IF GCS and U be an IFROS such that A U: Since every IFROS is an IFOS and A is an IF GCS, we have cl(A) U: Hence A is an IFG CS.

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Example 3.14. Let X = fa; bg and = f0~; G; 1~g	be an IFTS on X, where

G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS A = hx; (0:4; 0:5); (0:6; 0:5)i is an IFG CS but not an IF GCS in (X;):

Theorem 3.15. Every IFG CS is an IFG CS but not conversely.

Proof. Let A be an IFG CS and U be an IFROS such that A U: Since every IFROS is an IF OS and by hypothesis, we have cl(A) U: Hence A is an IFG CS.

Example 3.16. Let X = fa; bg and = f0~; G; 1~g be an IFTS on X, where

G = hx; (0:8; 0:8); (0:2; 0:1)i : Then the IFS A = hx; (0:9; 0:7); (0:1; 0:3)i is an IFG CS but not an IFG CS in (X;):

Remark 3.17. Summing up the above theorems, we have the following diagram. None of the implications are reversible.

IFCS ! IFGCS ! IFRGCS

% &

IFRCS IFG CS

& %

IF CS ! IFG CS ! IF GCS

Remark 3.18. The following examples show that IFG CS is idependent of

IFPCS, IFSCS, IF CS, IFGSCS and IFGSPCS.

Example 3.19. Let X = fa; bg and = f0~; G_1; G_2; 1~g be an IFTS on X, where

G_1 = hx; (0:7; 0:8); (0:3; 0:2)i and G_2 = hx; (0:6; 0:7); (0:4; 0:3)i : Then the IFS A = h x; (0:6; 0:8); (0:4; 0:2) i is an IFG CS but not an IFPCS, IFSCS, IF CS, IFGSCS and IFGSPCS in (X;):

Example 3.20. Let X = fa; bg and = f0~; G; 1~g be an IFTS on X, where

G = hx; (0:5; 0:4); (0:5; 0:6)i : Then the IFS A = hx; (0:4; 0:2); (0:6; 0:7)i is an IFPCS, IF CS, IFGSCS

and IFGSPCS but not an IFG CS in (X;):
Example 3.21. Let X = fa; bg and = f0~; G; 1~g be an IFTS on X, where

G = hx; (0:5; 0:4); (0:5; 0:6)i : Then the IFS A = hx; (0:5; 0:5); (0:5; 0:6)i is an IFSCS but not an IFG CS in (X;):

Theorem 3.22. The union of two IFG C sets is an IFG CS in (X;):

Proof. Let U be an IFROS in (X;) such that A[B U: Then A U and B U: So, cl(A) U and cl(B) U: Therefore cl(A) [cl(B) cl(A [B) U:

Hence A [B is an IFG CS.

Remark 3.23. Intersection of two IFG C sets need not be an IFG CS.

Example 3.24. Let			X	= fa; b; cg and		= f0~; G_1; G_2; G_1 [G_2; G_1 \ G_2; 1~g	
be	an IFTS	on	X,	where	G_1 =	hx; (0:0; 0:6; 0:1); (0:1; 0:25; 0:0)i	and
G_2	=	hx; (0:1; 0:25; 0:0); (0:0; 0:6; 0:1)i :			Then	the IFS A	=
hx; (0:3; 0:7; 0:1); (0:2; 0:25; 0:3)i				and the IFS B	= hx; (0:0; 0:6; 0:3); (0:1; 0:3; 0:0)i		
are IFG C sets but A \ B is not an IFG CS.							
Theorem 3.25. If an IFS A is an IFG CS such that						A Bcl(A); where	
B is an IFS in an IFTS (X;);				then B is an IFG CS in (X;):			

Proof. Let U be an IFROS in (X;) such that B U: Then A U: Since A is

an IFG CS, we have cl(A) U: Now, cl(B) cl(cl(A)) = cl(A) U:

Hence B is an IFG CS in (X;):

Theorem 3.26. If an IFS A is an IFRGCS such that A B cl(A); where B is an IFS in an IFTS (X;); then B is an IFG CS in (X;):

Proof.	Let U be an IFROS in (X;) such that B U: Then					A U: Since	
A is an IFRGCS and		cl(A) cl(A); we have cl(A)				cl(A) U: Now,	
cl(B)		cl(B)		cl(A)		U: Hence B is an IFG CS in (X;):	
Theorem 3.27. An IFS A is an IFG CS in an IFTS					(X;)	if and only if	
e(AqF)	implies e(cl(A)qF) for every IFRCS F of (X;):						

Proof. Necessity. Assume that A is an IFG CS in (X;): Let F be an IFRCS and e(AqF): Then A F ^{c}; where F ^{c} is an IFROS in (X;): Then by assumption,

cl(A) F ^{c}: Hence e(cl(A)qF):

Su ciency. Let F be an IFROS in (X;) such that F U: Then F ^{c} is an

IFRCS in (X;) and F $(U^c)^c$: By assumption, e(FqU^c) implies e(cl(A)qU^c):

Therefore, cl(A) $(U^c)^c$ = U: Hence A is an IFG CS in (X;):

Theorem 3.28. If A is an IFROS and an IFG CS in (X;); then A is an

IF CS in (X;):

Proof. Let A be an IFROS. Since A A; cl(A) A: But A cl(A) always. Therefore cl(A) = A: Hence A is an IF CS in (X;):

Theorem 3.29. Every IFS in an (X;) is an IFG CS if and only if IF OS and IF CS coincide.

Proof. Necessity. Suppose that every IFS in (X;) is an IFG CS. Let U be an IFROS in (X;): Then U is an IFOS and an IF OS and by hypothesis

cl(U) U cl(U): That is cl(U) = U: Thus U is an IF CS in (X;):

Hence IF O(X) IF C(X): Let A be an IF CS. Then A^c is an IF OS in (X;): But IF O(X) IF C(X): Therefore A is an IF OS in (X;); we have

IF C(X) IF O(X): Thus IF O(X) = IF C(X):

Su ciency. Suppose that IF O(X) = IF C(X): Let A U and U be an IFROS in (X;): Since every IFROS is IF OS, U is an IF OS in (X;) and therefore cl(A) cl(U) = U; by hypothesis. Hence A is an IFG CS in (X;):

Theorem 3.30. An IFS A of an IFTS (X;) is an IFROS and an IFG CS, then A is an IFRCS in (X;):

Proof. Let A be an IFROS and an IFG CS in (X;): Then	cl(A)		A:
Since cl(A) is an IF CS, we have cl(int(cl(A))) A: Therefore	cl(A)		A;

since A is an IFROS. Then cl(int(A)) cl(A) A: Therefore cl(int(A)) A:

Since every IFROS is an IFSOS, A is an IFSOS and we have A cl(int(A)): Thus

A = cl(int(A)): Hence A is an IFRCS in (X;):

Theorem 3.31. Let A be an IFG CS in (X;) and $p_{(;)}$ be an IFP in X such that cl(A) q cl($p_{(;)}$): Then A q cl($p_{(;)}$):

Proof. Assume that A is an IFG CS in (X;) and cl(A) q cl($p_{(;)}$): Suppose that e (A q cl($p_{(;)}$)); then A (cl($p_{(;)}$))^c where (cl($p_{(;)}$))^c is an IF OS

in (X;): Then by De nition 3.1, cl(A) int(cl(cl($p_{(;)}$))^c) cl(($p_{(;)}$

De nition 4.1. An IFS A of an IFTS (X;)	is called an IFG OS if and only

Therefore e (cl(A) q cl($p_{(;)}$)); which is a contradiction to the hypothesis. Hence A q cl($p_{(;)}$):

if A^c is an IFG CS.		
Theorem 4.2. Every IFOS, IFROS,	IF OS, IFGOS, IFRGOS, IF GOS,	
IFG OS is an IFG OS in (X;):		
Proof. Obvious.		
Example 4.3. Let X = fa; bg and	= f0~; G; 1~g be an IFTS on X, where	
G = hx; (0:5; 0:6); (0:5; 0:4)i : Then the IFS		A = hx; (0:6; 0:5); (0:4; 0:5)i is an
IFG OS but not an IFOS, IFROS, IF OS, IFGOS, IF GOS in (X;):		
Example 4.4. Let X = fa; bg and	= f0~; G; 1~g be an IFTS on X, where	
G = hx; (0:8; 0:8); (0:2; 0:1)i : Then the IFS		A = hx; (0:1; 0:3); (0:9; 0:7)i is an
IFG OS but not an IFG OS in (X;):		

Example 4.5. Let X = fa; b; cg and = f0~; G_1; G_2; 1~g be an IFTS on X, where

G_1 = hx; (0:4; 0:4; 0:5); (0:4; 0:4; 0:4)i and G_2 = hx; (0:2; 0:3; 0:5); (0:5; 0:5; 0:5)i : Then the IFS A = h x; (0:5; 0:4; 0:5); (0:4; 0:3; 0:2) i is an IFG CS but not an IFRGCS in (X;):

Theorem 4.6. An IFS A of an IFTS (X;) is an IFG OS if and only if

U int(A) whenever U A and U is an IFRCS.

Proof. Necessity. Assume that A is an IFG OS in (X;): Let U be an IFRCS such that U A: Then U^c is an IFROS and A^c U^c: Then by assumption A^c is an IFG CS in (X;): Therefore, we have cl(A^c) U^c: Hence U int(A):

Su ciency. Let U be an IFROS in (X;) such that A^c U: Then U^c A and U^c is an IFRCS. Therefore U^c int(A): Since U^cint(A); we have (int(A))^c U that is cl(A^c) U . Thus A^c is an IFG CS. Hence A is an IFG OS in (X;):

Remark 4.7. Intersection of two IFG O sets is an IFG OS in (X;): But the union of two IFG O sets need not be an IFG OS.

Example 4.8. Let X = fa; b; cg and = f0~; G_1; G_2; G_1 [G_2; G_1 \ G_2; 1~g be an IFTSon X, where G_1 = hx; (0:0; 0:6; 0:1); (0:1; 0:25; 0:0)i and G_2 =hx; (0:1; 0:25; 0:0); (0:0; 0:6; 0:1)i : Then the IFS A = hx; (0:2; 0:25; 0:3); (0:3; 0:7; 0:1)i and the IFS B = hx; (0:1; 0:3; 0:0); (0:0; 0:6; 0:3)i are IFG O sets but A \ B is not an IFG OS.

Theorem 4.9. Let A Be an IFS in (X;): If B is an IFSOS such that B A int(cl(B)); then A is an IFG OS in (X;):

Proof. Since B is an IFSOS, we have B cl(int(B)): Thus, A int(cl(B)) int(cl(cl(int(B)))) = int(cl(int(B))) int(cl(int(A))): This implies A is an IF OS: By Theorem 4.2, A is an IFG OS in (X;):

Theorem 4.10. If an IFS A is an IFG OS in (X;) such that int(A) B A; where B is an IFS in (X;); then B is an IFG OS in (X;):

Proof. Suppose that A is an IFG OS in (X;) and int(A) B A: Then A^c is an IFG CS and A^c B^c (int(A))^c; this implies A^c B^ccl(A^c): Then B^c is an IFG CS in (X;); by Theorem 3.26. Hence B is an IFG OS in (X;):

Theorem 4.11. If an IFS A is an IFRGOS in (X;) such that int(A) B A; where B is an IFS in (X;); then B is an IFG OS in (X;):
Proof. Let A be an IFRGOS and int(A) B A: Then A^c is an IFRGCS and
A^c B^c cl(A^c): Then B^c is an IFG CS in (X;); by Theorem 3.27. Hence B is an IFG OS in (X;):

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